Diplomhauptprüfung / Masterprüfung

System Dynamics and Control Engineering

March 20th, 2008

Problems

The solutions and the entire and comprehensible way to the solutions have to be written down in the provided solution sheets. Only those are taken into account. Use document proof writing utensils only.

At the beginning of the examination, write your name and matriculation number on the first page of the solution sheets.

Hand over all provided solution sheets after the examination.
Problem 1

In figure 1.1 a control loop is given

\[ G_1(s) = 1.3 \cdot \frac{(1+0.5s)}{(1+3s)} e^{-0.5s}, \quad G_2(s) = \frac{1}{2s} \quad \text{and} \quad G_3(s) = \frac{1}{1+0.5s}. \]

a) This system shall be controlled by means of a P-controller \( G_c(s) = K_C \) with \( K_C \in \mathbb{R}^+ \).

- Specify the transfer function of the open loop.
- Is it reasonable to use a P-controller in this case?
- Can the Hurwitz-criterion be used to test the closed loop stability?

Now the control loop depicted in figure 1.2 is given

\[ G_P(s) = \frac{1-s}{1+s} e^{-T_i s}, \quad T_i \in \mathbb{R}^+ \quad \text{and} \quad G_C(s) = K_C, \quad K_C \in \mathbb{R}_0^+. \]
b)  
- Calculate the open loop step response.
- Sketch the Nyquist plot of the system depicted in Figure 1.2 with $K_C = 1$ into the diagram provided in the solution sheets.
- For which values of $K_C$ is the closed loop stable?

Now the control loop depicted in Figure 1.3 is given

![Figure 1.3](image)

with

$$G_1(s) = \frac{0.5}{(1+4s)(1+2s)(1+0.5s)} \quad \text{and} \quad G_2(s) = \frac{0.1(1+s)}{(1+0.5s)(1+0.2s)}.$$ 

c) An ideal PID-controller shall be used as controller 1 for the inner loop.
- Determine the transfer function $G_{C1}(s)$ such that the closed inner loop is as fast as possible and shows the behavior of two identical PT1-elements which are serially connected?
- What is the value of the damping $d$ of the inner loop in that case?

The controller 1 just specified shall remain unchanged for the subsequent tasks.

d)  
- A PI-controller is now used as controller 2. Using frequency response curves, specify the controller parameters $(K_C, T_N)$, such that the closed loop is as fast as possible and a phase margin of $60^\circ$ is kept. Therefore, sketch the amplitude response (asymptotes) and the phase response (straight lines approximation) of the open loop, which is corrected by means of that PI-controller with $K_C = 1$ into the diagram provided in the solution sheets.
- Why is it possible to operate the overall system stable, although with
  controller 1: PID-controller
  controller 2: PI-controller
  two I-elements are connected in series, with each having a phase shift of -90° independent of the frequency?
Problem 2

Given is a control loop according to figure 2.1.

![Block diagram of the control loop](image)

Figure 2.1

At first for the transfer function of the plant

\[ G_p(s) = \frac{10}{s(s+4)(s^2+4s+8)} \]

holds.

a) This system shall be controlled by means of a P-controller \( G_C(s) = K_C \), \( K_C \in \mathbb{R}_0^+ \). For the controller design the root locus curves are to be used.

- Sketch the root locus curve for the given system into the diagram provided in the solution sheets. Calculate the root center of gravity, the inclination angles of the asymptotes, branching points and angles between neighboring curves at the branching points as well as the inclination angles at first. Furthermore, specify the directions in which the branches of the root locus curve are run through.

- Can the closed loop be unstable?

  If your answer is yes: Determine the eigenfrequency as well as the corresponding controller gain \( K_C \) (P-controller) for the system at stability limit.
b) This system shall now be controlled by means of a PI-controller.

Calculate the parameters for such a PI-controller using the tuning rules according to Ziegler-Nichols.

In the following, for the transfer function of the plant according to figure 2.1

\[ G_p(s) = \frac{10}{(1 + 5s)(1 + s)} \]

holds.

A controller shall be determined such that the overshoot \( \Delta m \) of the step response \( h(t) \) of the closed loop does not exceed

\[ \Delta m_{\text{max}} = \frac{h(t)\big|_{\text{max}} - h(\infty)}{h(\infty)}. \]

For that purpose, a PI-controller or a real PID-controller may be used in such a way, that the closed loop is as fast as possible.

c) Calculate the value of the step response \( h(t) \) for \( t \to \infty \) with a command step \( w(t) = \sigma(t) \)

if a P-controller with \( G_C(s) = K_C = \frac{1}{90} \) is used instead of one of the suggested controllers.

d) Calculate the controller transfer function for each of the suggested controller types, if the entire allowed maximum overshoot \( \Delta m_{\text{max}} = 10\% \) shall be used in order to achieve a control which is as fast as possible.

**Hint:**
- For the real PID-controller, the denominator time constant is to be chosen to be a tenth of the smallest controller time constant.
- \( \log_{10}(\Delta m) = -\frac{d}{\sqrt{1 - d^2}} \quad (d \triangleq \text{damping}) \)
e) Which one of the two suggested controller types belongs to the faster control loop?

Give reasons for your answer by calculating each time value

\[ t_m = \frac{\pi}{\omega_n \sqrt{1 - d^2}} \]

at which the step response \( h(t) \) reaches its maximum value.

f) The transfer function of the plant according to figure 2.1 is still supposed to be

\[ G_p(s) = \frac{10}{(1 + 5s)(1 + s)} \]

Using root locus curves, design a PI-controller such that it is as fast as possible and its damping shows a minimal value of \( d_{\text{min}} = 0.5 \).

Calculate and mark the area, in the diagram provided at the solution sheets, in which the poles of the closed loop may be located. Furthermore, mark the location of the poles at minimum damping \( d_{\text{min}} = 0.5 \) particularly. Calculate for this the maximum value for the gain of the PI-controller.
Problem 3

In figure 3.1 a three-way-catalyst is depicted schematically. Such catalysts are used in motor vehicles in order to reduce the pollutants of Otto engines at insufficient combustion. However, they are only effective, if they have reached a certain temperature – the so-called light-off temperature. Beneath this temperature, only an insufficiently small amount of pollutants can be converted. For that reason, catalytic converters feature an electrical heating in order to heat up the catalyst and the exhaust towards light-off temperature as fast as possible.

It is your task to design a controller for the heating system of the catalyst, which controls the exhaust temperature $T_{out}$ at output towards the corresponding reference temperature $T_{out,ref}$.

A temperature sensor, which provides a voltage $u_{meas}$ in dependence of the exhaust temperature, is mounted straight at the output of the catalyst.

Equations (1) to (5) represent the transformation of electrical energy towards heat energy of the exhaust. For the sake of simplicity, the exhaust mass flow rate $m_{flow}$, the thermal capacity $c_{p,g}$ of the exhaust and the input temperature $T_{in}$ of the exhaust are considered as constant.

Figure 3.1
• The exhaust flow enthalpy of the catalyst at input and output, respectively, are

\[ P_{in} = m_{flow} \cdot c_{p,g} \cdot T_{in} \]  

\[ P_{out} = m_{flow} \cdot c_{p,g} \left( T_{in} + T_H \right) \]  

with \( m_{flow} = 0.00015 \frac{kg}{s} \) being the exhaust mass flow rate and \( c_{p,g} = 1000 \frac{J}{kg \cdot K} \) being the specific thermal capacity of the exhaust.

The input temperature of the exhaust is \( T_{in} = 450K \).

• The electrical heating power is

\[ P_H = \frac{u_H^2}{R_H} \]  

with \( R_H = 15\Omega \) being the resistance of the heating coil conductor.

• The (differential) equations for the heat transfer from the heating coil to the catalysts housing and from the housing to the exhaust, respectively, are

heating coil → catalyst housing: \[ P_{housing} + K_{housing} \cdot \dot{P}_{housing} = P_H \]  

catalyst housing → exhaust: \[ P_{out} - P_{in} = P_{housing} \]  

with the heating constant \( K_{housing} = 2\sec \) of the catalysts housing.

• The characteristic curve of the temperature sensor was derived from measurements and is depicted in figure 3.2 (larger figure in the solution sheets).

![Figure 3.2](image-url)
Your first task is to model that system.

a) Sketch the signal flow diagram of that system using $u_H$ as input and mark the contained nonlinearities using a double border.

b) At the operating point (equilibrium point), the exhaust temperature at the output equals the light-off temperature, i.e. $T_{out,\theta} = 550K$.

- Calculate the values of all variables, at operating point.
- Which nonlinearities are contained within the system?

c) Linearize the system around the operating point.

- Sketch the signal flow diagram of the linearized system.

d) Which type of controller is suitable for the linearized system, if the closed loop should be stable and steady-state accurate for a step as input signal.
Problem 4

An electric circuit according to figure 4.1 is given which corresponds to a bridge circuit commonly used at measurement instrumentation.

Figure 4.1

a) Describe the behavior of the bridge circuit according to figure 4.1 in state-space representation using vector-matrix notation! As state variables, use the voltages $x_1 = u_1$ and $x_2 = u_2$. The provided voltage $u$ is regarded as input variable and the bridge voltage $y$ as output variable.

b) Calculate the value of $\alpha$ such that $0 = \frac{1}{2} u_0$ holds at steady-state.

c) • What are the eigenvalues of the system dependent on $\alpha$?
  • Sketch the course of the eigenvalues in dependency of $\alpha$ with $0 \leq \alpha \leq 1$ into the diagram provided in the solution sheets. Use the following normalized values:

  $$ R_1 = 2 \quad , \quad R_2 = 1 \quad , \quad R_p = 5 \quad , \quad C_1 = \frac{1}{2} \quad , \quad C_2 = \frac{2}{5} \quad . $$
The following system is considered for the subsequent tasks:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 1 \end{bmatrix} x .
\end{align*}
\]

d) \hspace{1em} \bullet \text{ Verify, whether that system is controllable.} \\
\hspace{1em} \bullet \text{ Specify the controller parameters of a state-feedback controller } u = - \begin{bmatrix} r_1, r_2 \end{bmatrix} x \text{ using Ackermann’s formula, such that the controlled system has eigenvalues at } \\
\lambda_{C,1} = \lambda_{C,2} = -2 .

e) \hspace{1em} \bullet \text{ Verify, whether that system is observable.} \\
\hspace{1em} \bullet \text{ Design an observer for this system which shows a double eigenvalue at } \\
\lambda_{O,1} = \lambda_{O,2} = \alpha . \text{ Calculate the vector } k_O \text{ and specify the state equations of the observer.} \\
\hspace{1em} \bullet \text{ What are reasonable values for the double eigenvalue } \alpha \text{ of the observer.}